CBCS SCHEME

USN 4VP21AI051 | 21MAT11

First Semester B.E./B.Tech. Degree Examination, Feb./Mar. 2022
Calculus and Differential Equations

Time: 3 hrs. | Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. With usual notation prove that $\tan\phi = r\dfrac{d\theta}{dr}$. **(06 Marks)**

b. Find the angle between the curves $r = a\log\theta$ and $r = \dfrac{a}{\log\theta}$. **(07 Marks)**

c. Find the radius of curvature for the cardioid, $r = a(1+\cos\theta)$. **(07 Marks)**

OR

2 a. With usual notation prove that $\rho = \dfrac{(1+y_1^2)^{3/2}}{y_2}$. **(06 Marks)**

b. Show that $r = 4\sec^2\theta/2$ and $r = 9\,\mathrm{cosec}^2\,\theta/2$ the pair of curves cut orthogonally. **(07 Marks)**

c. Find the pedal equation of the curve $r^n = a^n\cos n\theta$. **(07 Marks)**

Module-2

3 a. Expand $\sqrt{1+\sin 2x}$ by Maclaurin's series up to the term containing x^4. **(06 Marks)**

b. If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

c. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$ find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, -1, 0). **(07 Marks)**

OR

4 a. Evaluate $\lim\limits_{x\to 0}\left(\dfrac{a^x + b^x + c^x}{3}\right)^{1/x}$. **(06 Marks)**

b. If $z = e^{ax+by} f(ax - by)$ prove that $b\dfrac{\partial z}{\partial x} + a\dfrac{\partial z}{\partial y} = 2abz$. **(07 Marks)**

c. Find the extreme values of $f(x,y) = x^3 + y^3 - 3x - 12y + 20$. **(07 Marks)**

Module-3

5 a. Solve $\dfrac{dy}{dx} + \dfrac{y}{x} = y^2x$. **(06 Marks)**

b. Find the orthogonal trajectories of the family of curves $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2+\lambda} = 1$, where λ is a parameter. **(07 Marks)**

c. Solve $x(y')^2 - (2x+3y)y' + 6y = 0$. **(07 Marks)**

OR

6 a. Solve $(x^2 + y^2 + x)dx + xydy = 0$. **(06 Marks)**

b. If the temperature of the air is 30°C and a metal ball cools from 100°C to 70°C in 15 minutes, find how long will it take for the metal ball to reach a temperature of 40°C. **(07 Marks)**

c. Find the general solutions of $xp^2 + xp - yp + 1 - y = 0$. **(07 Marks)**

Module-4

7 a. Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$. **(06 Marks)**

b. Solve $(D^3 + D^2 - 4D - 4)y = 3e^{-x}$. **(07 Marks)**

c. Solve $\dfrac{d^2y}{dx^2} + y = \sec x \tan x$ using the method of variation of parameters. **(07 Marks)**

OR

8 a. Solve $(D^2 + 4)y = x^2$. **(06 Marks)**

b. Solve $\dfrac{d^2y}{dx^2} - 4y = \cosh(2x - 1)$. **(07 Marks)**

c. Solve $(x^2D^2 + xD + 9)y = 3x^2$. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix.

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}.$$
(06 Marks)

b. Solve by Gauss elimination method

$2x + y + 4z = 12$
$4x + 11y - z = 33$
$8x - 3y + 2z = 20$. **(07 Marks)**

c. Solve the system of equation by Gauss-Seidel method

$20x + y - 2z = 17$
$3x + 20y - z = -18$
$2x - 3y + 20z = 25$. **(07 Marks)**

OR

10 a. Find the values of λ and μ such that the system of equations:

$x + y + z = 6$
$x + 2y + 3z = 10$
$x + 2y + \lambda z = \mu$, may have

i) unique solution ii) infinite solution iii) no solution. **(06 Marks)**

b. Solve by the method of Gauss-Jordan method:

$2x + 5y + 7z = 52$
$2x + y - z = 0$
$x + y + z = 9$. **(07 Marks)**

c. Find the largest eigen value and the corresponding eigen vector of the matrix

$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ by using the power method by taking initial vector as $[1, 1, 1]^T$.

(07 Marks)

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